Mathematics for Physics: Errata

August 17, 2022

p8: A ring unit is defined for unital rings, not commutative rings.

p8: Finite fields have order p^n with p prime, but only fields of prime order are of the form $\mathbb{Z}_p$; the last bullet should be replaced with:

- Every finite field (AKA **Galois field**) has order p^n with p prime (denoted $GF(p^n)$ or $\mathbb{F}_{p^n}$), and is unique (up to isomorphism)
- $GF(p)$ is isomorphic to $\mathbb{Z}_p$, the integers modulo p (also denoted $\mathbb{Z}/p\mathbb{Z}$ or $\mathbb{Z}/\left(p\right)$)

p13: The signature is the number of positive and negative values of the orthonormal basis vector inner products, not of their "lengths," and is independent of the choice of *orthonormal* basis.

p16: Division algebras as we have defined them are finite-dimensional unital division algebras.

p21: The list of isomorphisms at the end of the section are as real algebras.

p24: An ideal is usually defined to also include the entire ring (a prime ideal usually remains a proper ideal). Note that if a ring is defined to include a unity, an ideal is not necessarily a subring.

p29: In stating that $A = a\hat{e}_1 \wedge \cdots \wedge \hat{e}_k$ implies $\langle A, A\rangle = \pm a^2$, the e_μ are assumed orthonormal and should have been denoted $\hat{e}_\mu$.

p39: In the steps showing $\varphi(v) = \left\langle \varphi^\sharp, v\right\rangle$, the third line should omit $\langle e_\mu, e_\lambda\rangle$ to read $= \eta_{\mu\lambda}\varphi^\lambda v^\mu$.

p40: The last paragraph of Section 3.2.2 was omitted in the print run; this paragraph is what is cross-referenced in Section 3.1.1 on p27 after the definition of the tensor algebra:

> The infinite direct sum of the tensor spaces of every type forms an associative algebra. This algebra is also called the "tensor algebra," and "tensor" sometimes refers to the general elements of this algebra, in which case tensors as defined above are called **homogeneous tensors**. In this book, we will always use the term "tensor" to mean homogeneous tensor, while for "tensor algebra" the inclusion of powers of the dual space will depend upon context.

p43: For an order 2 tensor to operate as a linear mapping on vectors and forms, it must be of type $(1, 1)$.

p46: The relationship between the two inner products should be $\langle \bigwedge \varphi_i, \bigwedge \psi_j\rangle_{\text{tensor}} = k!\det\left(\langle \varphi_i, \psi_j\rangle\right)$.

p49: If we consider a pseudo inner product with signature (r, s), the last equation should be $i_v\Omega = (-1)^s * (v^\flat)$.

p76: The Betti number should be denoted b_n, and is not the number of n-cells in a cell complex, it is the number of holes of dimension n.

p80: The definition of relative homotopy groups should read $s_0 \in \partial D^n$ and $x_0 \in A \subset X$, and are only defined for $n > 1$.

p85: $f \circ \alpha^{-1}\colon \mathbb{R} \to \mathbb{R}$ should be $f \circ \alpha^{-1}\colon \mathbb{R}^n \to \mathbb{R}$.

p91: The torus (with any number of holes) is the only closed orientable surface with a non-zero smooth vector field.

p95: The Whitney embedding theorem is valid for positive codimension.

p95: A regular point is where $\mathrm{d}\Phi_p$ maps T_pM onto $T_{\Phi(p)}N$.

p110: The cube is dual to the octahedron, not the tetrahedron (figure caption and following text).

p120, second warning: A as a vector field on $\mathbb{R}^n$ should not be confused with A as a vector field on GL; gl is not a "vector field of vector fields."

p121: $(\exp(A))^\dagger$ = -$\exp(A^T)$ should read $(\exp(A))^\dagger = \exp(A^\dagger)$.

p130: An intertwiner must be *defined* to be linear, this is not implied by equivariance.

p134: Any *complex* irrep of an abelian Lie group is one-dimensional (as a complex manifold).

p143: The complexified $C(r,s)$ is equivalent to the Clifford algebra generated by $\mathbb{C}^n$ under the *real* inner product $\langle v,w\rangle \Rightarrow \langle v,v\rangle \in \mathbb{C}$.

p143: Only completely reducible reps of $\mathbb{K}(n)$ are direct sums of the irreducible ones.

p143-144: The faithful reps which are direct sums split into two irreps which are *not* faithful.

p146: Dirac matrices act as an orthonormal basis of the vector space generating $C(3,1)$ or $C(1,3)$ (and via complexification $C^{\mathbb{C}}(4)$).

p150: The elements $U \in \mathrm{Pin}\,(r,s)$ are those which satisfy $U\widetilde{U} = \pm 1$, where U is still assumed to be a product of invertible vectors per the rotation construction.

p152, first sentence: $\mathrm{Pin}\,(r,s)$ and $\mathrm{Spin}\,(r,s)$ are always double covers of $O(r,s)$ and $SO(r,s)$, including when $r=s=1$.

p155: We can drop the $\pm$ factor since $-\exp(i\theta) = \exp(i(\theta - \pi))$.

p157: With a $(1,3)$ signature, the mixed bivector reps remain the same, but the spacelike ones flip sign; thus U_R and U_L are not swapped, and the warning should omit the equation and begin "In mapping the above..." with the correction "...Lorentz transformation reps *remains the same*...".

p158: The Weyl irreps are not faithful.

p159, para 2: The rep of $C_0(1,3) \cong C(3,0)$ is effected by $\hat{e}_i\hat{e}_0 \to \sigma_i$, and thus $x\hat{e}_0 = x^0 + x^i\sigma_i$ to preserve the pseudo-scalar; for mostly pluses the space-time split should use $-x\hat{e}_0$ with $\hat{e}_0\hat{e}_i \to \sigma_i$.

p160, para 1: Last sentence should not be present; the alternative Weyl rep is not equivalent to the standard Weyl rep (as stated in the next para).

p170: Typo in the exponential expansion $\lim_{\varepsilon\to 0}(1+\varepsilon x)^{1/\varepsilon}$.

p190-2: "parallel translate" should be "parallel transport" throughout. The last construction requires zero torsion in order that the parallel transports form a parallelogram; the last figure with non-zero commutator is thus equivalent to non-zero torsion, since the two are negatives of each other.

p194: The minimum length curve *defines* a (Riemannian) geodesic; there is a unique such geodesic whose tangent is a given vector, which then defines the exponential map.

p195-6: The definition of Riemann normal coordinates should have been placed in the following section on the Levi-Civita connection, where zero torsion and a metric connection ensure the partials of the metric vanish.

p199-200: For a pseudo-Riemannian manifold of signature (r,s), $\mathrm{div}(u) \equiv (-1)^s * \mathrm{d}(*(u^\flat))$. Using the previously stated relations $i_u\Omega = (-1)^s * (u^\flat)$ and $A = (*A)\Omega$ for $A \in \Lambda^n M^n$, we have $\mathrm{d}(i_u\Omega) = (-1)^s\mathrm{d}(*(u^\flat)) = (-1)^s * \mathrm{d}(*(u^\flat))\Omega = \mathrm{div}(u)\Omega$.

p201: The coordinate expressions in the table should be $\mathrm{j}^1\mathrm{d}^2x$, where d^3x are coordinates with x^1 constant on S and normal to it, and the current density's value is based on the vector's coordinate length.

p205: The definition of G in terms of the sectional curvature is missing a factor $g_{\mu\mu}$, so that $G(\hat{v},\hat{v})$ is $-\langle\hat{v},\hat{v}\rangle(n-1)(n-2)/2$ times the average of the orthogonal sectional curvatures. This also is incorrect on p214, 9.3.8 Summary.

p218, last box: the reference to the next section should be to the section defining fiber bundles.

p221, figure: the spinor reps are on $\mathbb{K}(m)$, and the $u(n)$ rep is $-iqA^\alpha{}_\beta(v)$; the frame should be depicted such that the field components at $p+\varepsilon v$ are those of the field value at p applied to the frame at $p+\varepsilon v$.

p222: A fibered manifold is defined when M and E are manifolds.

p254: The change of frame indices should read $\frac{\partial}{\partial x'^{\mu}_i} = \frac{\partial x^\lambda_i}{\partial x'^{\mu}_i}\frac{\partial}{\partial x^\lambda_i}$.

p259: The text "where the $U(1)$ factor is ignored" should be removed.

p259: The first paragraph of 10.5.1 should end with $\pi(\Phi(q)) = f(\pi_f(q)) = x \in M$.